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Challenges in Understanding the Emergence of Size Sequencing Skills: Models, Structures, and Educational Tools

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We thank the authors who wrote commentaries on our SRCD Monograph, *The Development of Size Sequencing Skills: An Empirical and Computational Analysis* (McGonigle-Chalmers & Kusel, 2019). These commentaries reveal colleagues' deep reflection on the monograph by challenging the assumptions of our computational modeling framework (Braithwaite, 2019), interrogating how Piagetian structuralism may relate to information processing accounts of cognitive change (Witherington, 2019), and highlighting broader educational implications of size sequencing skill development (Clements, 2019).

Reply to Braithwaite (2019)

Braithwaite's (2019) commentary is divided into two arguments. Firstly, he provides an interpretation of our theoretical position as that of "strategy discovery," and then discusses the implications of our having adopted this position, including an underspecification of computational mechanisms of change. Secondly, he proposes that a Working Memory (WM) capacity increase should be considered as a null hypothesis in explaining the discontinuity data we report, in advance of proposing possibly redundant representational artifacts as part of an explanation.

Regarding Braithwaite's (2019) first argument, our intended meaning of the term "discovery" in the monograph is not same as that presented by theorists such as Piantadosi et al. (2012) and Siegler and Araya (2005). These theorists' accounts of "strategy discovery" involve the recombination of sets of primitive "cognitive operators" into novel structures, thus allowing the expression of novel behaviors. By contrast, our use of the term "discovery" is restricted to information processing underlying the emergence of representations in Long-Term Memory (LTM) which do not involve such restructuring.

Before elaborating on this point, let us clarify our theoretical position. We see size sequencing skill as emerging from representational change in LTM, such change resulting from the directional elaboration and refinement of a set of core *design primitives*, alongside an increase in WM availability. These design primitives are built into the complex biological system's wetware, and can be implemented computationally as information processing mechanisms to test cognitive developmental theories (see McGonigle-Chalmers & Kusel, 2019, Chapter III; see also Boden, 1980). This is a different position from the strategy discovery stance as defined by Braithwaite (2019), in that we are not proposing a mechanism for the generation of *novel* representations. Rather, we float a mechanism for representational *growth* and *refinement*. This mechanism is seeded in baseline procedures that collaborate in a very specific way. Our null hypothesis as to size sequencing skill emergence thus relies heavily on a WM and LTM dynamic; we do not see changes to WM architecture alone as a feasible null hypothesis, as does Braithwaite (2019).

Our use of the term "discovery" in the monograph refers to the refinement of an emergent LTM structure that occurs during the operation of the transitional model, which is an extension of the heuristic model. A pre-existing bias towards "smallness" (the list of size differences within the denominator of equation 3b) is combined with an increase in WM availability. This allows the opportunity for size difference data to be meaningfully inspected by the agent. The agent subsequently notices that size difference minimality is an indicator for successful stimulus selection. This knowledge is persisted in the form of strengthening links between the adjacent elements of an emergent probabilistic rank order (the list of Beta distributions within the numerator of equation 3b). This is refinement in the sense that a pruning of a dominance hierarchical graphical structure (e.g., $A > B > C > D > E$) takes place, resulting in a chain graphical structure (e.g., $A\ r\ B\ r\ C\ r\ D\ r\ E$, where *r* signifies the smallest difference relation).

So, here we are not discovering new representations in the variation, evaluation, and selection sense defined by Braithwaite (2019), but iteratively refining the same representation in LTM, such that a 'steady state' is ultimately reached. On the links reaching a threshold value, and the Beta distributions reaching a threshold precision (their steady-state), slots form. These slots function as rules for "smallest difference" detection and containers for tallying (see McGonigle-Chalmers & Kusel, 2019, Chapter V). Slots are our source of discontinuity and are brought into operation in the principled search and ordinal models.

Aspects of our models are thus "hard coded," as noted by Braithwaite (2019). However, we feel that the hand-crafted aspects are needed to implement our theory of representational growth, which involves explicitly programming the functionality of a set of core design primitives. Moreover, our focus is on small sets of square stimuli varying only by one dimension at a time (size or color); our transactionalist stance means that the agent's LTM adapts to this state space. The format and function of the resulting steady-state of LTM (a unidirectional, chain-like structure) suggested to us the concept of a set of slots. A more complex sequential domain requiring logico-mathematical knowledge (such as non-monotonic size sequencing) with a wider space of possible representational formats may demand a different modeling approach. Strategy discovery would surely be a valuable modeling option to consider in this scenario.

The procedures in our heuristic model (SCAN, SCORE, HEURISTIC SELECT, INFER RANK and INHIBIT) are perhaps analogous to the cognitive operators noted by Braithwaite (2019) as requiring our specification. However, we do not see these procedures as requiring recombination to achieve optimality. We see them as *already* optimally configured in the young child, having been 'fossilised' into the genes by natural selection, expressed by developmental processes within the central nervous system, and ultimately providing possibilities for representational growth.

In terms of unspecified transitional mechanisms between models, we do specify mechanisms of change (albeit in a highly simplified way). For us, the crucial representational changes occur between the heuristic and principled models, which are specified within the transitional model. Moreover, the procedures defined in our models have much commonality, and could have been presented as one model, but for transparency concerns. However, the competition between the HEURISTIC SELECT with the PRINCIPLED SELECT procedures, and the extinction of the SCORE and INFER RANK procedures (which we describe on p. 80 in the monograph, as cited by Braithwaite, 2019) is undoubtedly underspecified, and a more realistic model could address such concerns. Indeed, our models are intentionally as simple as possible, and could be improved in faithfulness and plausibility. For example, at this point our models do not represent dynamics beyond those underlying aspects of representational emergence (see McGonigle-Chalmers & Kusel, 2019, p. 107 and p. 163). (See Sirois et al. [2008] for some constructivist views on transitional mechanisms.)

Regarding Braithwaite's (2019) second argument, we represent WM as a buffer of a fixed length of seven in all simulations, with a change to its architecture in terms of availability being introduced in the transitional model. It is unclear to us how changes to a buffer could by themselves allow the sudden emergence of rules of the format "if *X* then do *Y*." We see WM as a temporary store of information, and it follows that anything represented within it disappears between tasks. An LTM trace of some sort, beyond a temporary store, is surely needed to represent the emergence the "select smallest size difference" rule that drives the principled size sequencing model. (For further reading in this area, Miyake & Shah [1999] present many competing WM accounts, including those that emphasise the role of LTM.)

Based on our current understanding of Braithwaite's (2019) meaning of WM, we argue that theoretical accounts of size sequencing emergence that include the interaction of WM and LTM (also an aspect of the ACT-R and SOAR cognitive architectures) are still a necessity. Reflecting our stance on WM, Inhelder and Piaget's (1964) position on maturational factors affecting the development of seriation was as follows:

> . . . the maturation of the nervous system can do no more than create the conditions for a continual expansion of the field of possibilities. The realization of these possibilities demands not only the action of the physical environment (practice and acquired experience), but also the educational influences of a favourable social environment. (Inhelder & Piaget, 1964, p. 5)

Reply to Witherington (2019)

The challenge of how best to interpret Piaget's theoretical account was the subject of Witherington's (2019) commentary, specifically the tension he sees between structural and functional readings of the Piagetian literature, and the impact this has on information processing accounts of developmental change.

Of relevance here is that Boden (1980, quoting directly from Piaget's *Structuralism* text) noted Piaget's affinity for reality over formalism in the context of cybernetics:

> . . .one of the most instructive methods for analyzing [*the epistemic subject's--MB*] actions is to construct, by means of machines or equations, models of artificial intelligence for which a cybernetic theory can then furnish the necessary and sufficient conditions; what is being modeled in this way is not its structure in the abstract (algebra would suffice for this), but its effective realization and operation. (Piaget, 1970, p. 69 as quoted by Boden, 1980, pp. 136-137)

However, cybernetic formalisms did not lend themselves to such engineering (Boden, 1980). As Witherington (2019) notes, dynamical systems theory (DST) is well suited to modeling such cognitive change and has the benefit of being applicable to structural accounts of cognition. Complementary to this stance, we argue for the compatibility of suitably grounded information processing accounts of cognitive change and DST (McGonigle-Chalmers & Kusel, 2019, pp. 163-166), such that the benefits of both modeling approaches may be realized.

The debate as to the value of a formal structural interpretation of Piaget's position will no doubt continue. Our position is simply this: if Piaget himself believed that children learn from acting in and on the world—for which we cite many examples—then we need to specify what those actions are and how they can lead to representational change. If this can be done and be shown to lead to the same behavioral outcomes as described by Piaget without recourse to his structural concepts, then the explanatory value of the latter must be questioned. Couched in terms of changes in LTM content brought about by acting in the world, we were not applying a functionalist "reading" of Piaget's structuralism, but an analysis of what Piaget himself insisted to be the necessary antecedents to seriation success—behaving and acting on the world and reflecting on its consequences. We would argue that the resulting "powers" conform precisely to Witherington's (2019) criteria; they carry critical explanatory weight and can describe the agent's new "potential" regarding similar problems. We would even claim that emerging representation of a monotonic series ***is*** a structure in that it conforms to the most informationally lean representation of multiple size relations including their ordinal specification.

Reply to Clements (2019)

The commentary by Clements (2019) is on educational implications of size sequencing as causal to much cognitive change. He writes:

> McGonigle-Chalmers and Kusel's strong claim that multiple areas of reasoning emerge from the act of sequencing adds impetus to educational researchers' need to test, as well as build on, their findings and conclusion.

We appreciate and concur with this point, and indeed one of us (McGonigle-Chalmers) has floated the hypothesis that a virtual environment that allows the visualization of the products of successful size sequencing may accelerate cognitive achievements in both young children and non-human primates (see McGonigle-Chalmers & Kusel, 2019, pp. 176-177).

Concluding remarks

We thank the commentators again, and hope that this exchange will facilitate continuing debate of the theoretical, methodological, and educational issues raised in the monograph.

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