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Size Sequencing:
Increasingly Important for Theory, Research, and Practice

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Despite a long-standing interest in young children's development of basic cognitive competencies and their learning of mathematics, my initial reactions to the title of *The Development of Size Sequencing Skills: An Empirical and Computational Analysis* (McGonigle-Chalmers & Kusel, 2019) could be characterized as skeptical. My skepticism was twofold. First, I doubted that the focal topic was worthy of an entire monograph, and second, I doubted that it would attract a sufficiently broad readership for this journal. Reading just the introductory text, however, dispelled these concerns. The authors cogently argue that size-sequencing is still a core cognitive competence worthy of study by developmental scientists. In their monograph, they show that an historical and empirical investigation of children's seriation competence reveals not only a gap in recent research in this domain, but also an unfortunate theoretical divide which has important implications for other cognitive domains more broadly.

More specifically, Maggie McGonigle-Chalmers and Iain Kusel show how a bifurcation between Piagetian (Piaget & Szeminska, 1941/1952) and newer approaches to theory and methods in the study of cognition have led to a (false) dichotomy between perspectives. That is, this dichotomy has created argumentation that has been explicitly or implicitly based on the reasoning that any new advances (information processing, certain branches of cognitive science, neuropsychology, etc.), when supported with evidence, must be understood to *refute* a Piagetian position. The authors do not start with this dichotomous view and indeed present evidence that supports a dialectical synthesis.

As an example, they present empirical evidence that there is a developmental discontinuity in the way children respond to specific logical-mathematical tasks. This discontinuity appears to support the Piagetian position, or at the very least, to support Piagetian observations, which were too often the baby thrown away with the theoretical bathwater in the tradition of refutational research. Their new work offers a computationally supported correction to some information-processing approaches that emphasized mainly incremental accretions. Importantly, McGonigle-Chalmers and Kusel's computational model includes multiple cognitive components consistent with research in cognitive science, making it psychologically more plausible than earlier (mostly procedural) instantiations. And, of course, the combination of running a well-defined computational model and comparing results to studies with children

provides much-needed specificity to the model as well as triangulation to strengthen the model's usefulness (although one can always strive for greater levels of detail).

McGonigle-Chalmers and Kusel's work offers insights that should contribute not only to developmental theory and research, but also to educational practice. Their work addresses the issue perhaps best highlighted by the famous article entitled: "Either we're too early and they can't learn it or we're too late and they know it already: The dilemma of 'applying Piaget'" (Duckworth, 1979). The *Monograph* authors describe the benefits others have found in seriation training, and note that their research allows a more precise way both to conceptualize the possible nature of such training, and to identify ages at which such training is most likely to be beneficial, thus addressing Duckworth's dilemma.

The authors document that seriation training also has been conducted in an attempt to increase not only children's general cognition, but also their overall mathematics achievement. Positive findings from such efforts (e.g., Pasnak, Madden, Malabonga, Holt, & Martin, 1996) could be viewed as evidence of far transfer. That is, a general cognitive skill, seriation, is trained and one result is increased competence in the distinct domain of mathematics. There may be some truth in this view, that teaching general cognitive skills first promotes later learning of academic content. However, developmental psychologists and educators should also consider an alternative view. I propose that it may be less of a burden on crowded curricula and busy teachers to integrate seriation training into extant mathematics curricula. This has a dual advantage. Not only would it develop seriation skill, but it would honor the oft-neglected central role that seriation—writ large—plays in mathematics, from number and arithmetic, to geometry and measurement, to logic and reasoning. That role is arguably not recognized by many curriculum authors and teachers. Thus, a useful alternative to separate training, consolidating seriation and other mathematics experiences, may both elucidate that role for teachers and effectively teach both cognitive skills and mathematics content.

An example of this alternative approach is our *Building Blocks* preschool curriculum (Clements & Sarama, 2007/2013), which addresses seriation across multiple topics. *Building Blocks* is grounded in research-based learning trajectories for each of these topics (Clements, 2007; Clements & Sarama, 2009) and studies such as McGonigle-Chalmers and Kusel's are fundamental to this empirical base. I will elaborate on this last point before turning back to seriation examples. A learning trajectory for each specific topic has three components: (a) a goal, (b) a developmental progression of levels of thinking, and (c) instructional activities correlated to each level. To attain a specified mathematical competence in a given topic or domain (the goal), students learn each successive level (the developmental progression), aided by tasks (instructional activities) designed to build the mental actions-on-objects (concepts and procedures that act on them) that enable thinking at each higher level. The developmental progression of levels of thinking is based on research from developmental scientists and mathematics-education researchers. The instructional activities are designed to help children at level n construct the concepts and procedures that are required by level $n + 1$ by engendering those concepts and procedures. They do this in part by including external objects and actions that mirror the hypothesized mathematical activity of children as closely as possible. For example, objects may be shapes or sticks, and actions might be creating, copying, uniting, disembedding, and ordering both individual units and composite units. Tasks require children to apply— externally and then mentally—the actions and objects of the goal level of thinking (see

Clements & Battista, 2000, for further description and examples of mathematical objects/concepts and mathematical actions/processes that operate on them). In the following paragraphs, I provide examples of these learning trajectories relevant to seriation from *Building Blocks*.

The school topic that is perhaps the most obviously linked to seriation is measurement. At an early level of thinking, children learn to physically align two objects to determine which is longer or if they are the same length (Clements & Sarama, 2014; Sarama & Clements, 2009, see also LearningTrajectories.org). Thus, the mental objects are linear extents from endpoint to endpoint on each, and the actions on these objects are procedures that directly compare these extents. With this anticipatory scheme and perceptual support, children can compare the length of objects by physically sliding and rotating them into alignment (at one endpoint) and compare their other endpoints. An illustrative activity is, *How long is my arm*? in which children first cut pieces of string that match the lengths of their own arms, and then find things that are the same length, shorter, or longer.

At more advanced levels, children are asked to seriate physical objects such as sticks or "trains" of connecting cubes by length. The desired scheme is organized in a hierarchy, with the higher-order concept a (possibly implicit) image of an ordered series. Early actions-on-objects that estimate relative lengths (driving a trial and error approach) are eventually complemented by a scheme that considers each object in such a series to be longer than the one before it and shorter than the one after it (resulting in a more efficient strategy). For example, in the activity called *Build Cube Stairs,* children are asked to build a sequence of steps to help a toy animal reach her babies. The staircase begins with only three steps, and progressively grows to six or more. Children are also challenged to order a series of sticks by length, and then insert another stick that the teacher "forgot." Children are likewise given experience in ordering area and weight in a similar sequence.

The curriculum also targets seriation in the realm of number. Briefly, in the learning trajectory for comparing number, children develop an early level by perceptually evaluating which group of objects has more, when one of two groups has about double the number of the other. At a higher level, they are asked to compare the number in two groups of objects by counting and using the counting sequence to make a precise determination (e.g., "This is more because 9 comes after 8 when we count"). These abilities are extended to an even higher level of thinking with activities such as "Order Cards," in which children seriate cards with dots in sets of 1 to 5, and later, 1 to 10. They also work in pairs, taking turns hiding one card of the seriated set and figuring out which number is missing.

In these ways, seriation is taught along with specific mathematics content. Evaluations of the effectiveness of this curriculum show not only the success of this approach overall, but also suggest that the more traditional preschool curricula to which *Building Blocks* was compared do not similarly or adequately address seriation. That is, children in the *Building Blocks* group made the greatest relative gains on topics related to seriation (Clements & Sarama, 2007).

McGonigle-Chalmers and Kusel's strong claim that multiple areas of reasoning emerge from the act of sequencing adds impetus to educational researchers' need to test, as well as build on, their findings and conclusion. In particular, their work provides rich details to support core

components of learning trajectories for related topics that are illustrated by the curriculum I described briefly above and is described in more detail elsewhere (Clements & Sarama, 2014, 2017/2019; Sarama & Clements, 2009). That is, McGonigle-Chalmers and Kusel's work will contribute to the refinement and extension of each of these, most notably the developmental progression, but also the goal and instructional components.

We believe that the more precisely the hypothesized mental actions-on-objects are described, the better for educational researchers and other educators, for two reasons. First, assessment and diagnosis are improved, as is professional development that helps teachers understand and "see" the levels in children's behaviors ("teacher noticing," Superfine, Fisher, Bragelman, & Amador, 2017). Second, instruction is more effective the closer it mirrors accurate cognitive models and follows instructional implications such as those McGonigle-Chalmers and Kusel present. Much work needs to be done for direct contributions to praxis (especially given the likely cross-fertilization between seriation and other competencies), but the questions and methods are now clearer.

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